Roll No. | | | | | | | | | | |

ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

B.E./B.Tech. (Full Time) - END SEMESTER EXAMINATIONS, APRIL/MAY 2024

(Common to IBT, FOOD & PHARMA)

Semester IV

MA5354 PROBABILITY AND STATISTICS

(Use of Statistical tables may be permitted)

(Regulation 2019)

Time: 3 hours | Max. Marks: 100

CO 1	To analyze the performance in terms of probabilities and distributions achieved by the determined solutions
CO 2	To be familiar with some of the commonly encountered two dimensional random variables and be equipped for a possible extension to multivariate analysis
CO 3	To apply the basic principles underlying statistical inference
CO 4	To demonstrate the knowledge of applicable large sample theory of estimators
CO 5	To obtain a better understanding of the importance of the methods in modern industrial processes

BL – Bloom's Taxonomy Levels

(L1 - Remembering, L2 - Understanding, L3 - Applying, L4 - Analyzing, L5 - Evaluating, L6 - Creating)

PART- A (10 x 2 = 20 Marks)

(Answer all Questions)

Q. No	Questions	Marks	CO	BL
1	The random variable X has the p.m.f $p(x) = \begin{cases} \frac{1}{n} ; x = 1,2,3 \dots n \\ 0; \quad otherwise \end{cases}$. Compute $E(X)$.	2	1	L2
2	Let X be a geometric random variable with p.m.f $f(x) = \frac{3}{4}\left(\frac{1}{4}\right)^{x-1} ; x = 1,2,3 \dots$. Find the probability distribution of the random variable $Y = X^2$.	2	1	L2
3	The joint p.d.f of two dimensional random variable (X,Y) is given by $f(x,y) = \begin{cases} k(x^2+y^2); 20 \le x \le 30, 20 \le y \le 30 \\ 0 ; \quad otherwise \end{cases}$. What is the value of k.	2	2	L2
4	State central limit theorem.	2	2	L1
5	Define level of significance in testing of hypothesis.	2	3	L1
6	Write down the value of χ^2 for a 2×2 contingency table with cell frequencies 83, 57,45 and 68.	2	3	L3
7	Fill in the blanks: <table><tr><td>Source of variation</td><td>Degree of freedom</td><td>Sum of squares</td><td>Mean sum of squares</td><td>F ratio</td></tr><tr><td>Treatments</td><td>........</td><td>540.69</td><td>.......</td><td>F = ...</td></tr><tr><td>Error</td><td>12</td><td>........</td><td>7.15</td><td></td></tr><tr><td>Total</td><td>15</td><td>626.44</td><td></td><td></td></tr></table>	2	4	L2
8	Write any two assumptions of ANOVA test?	2	4	L1
9	What is meant by acceptance sampling in Statistical Quality Control?	2	5	L1
10	Write the control limits for fraction of defective chart.	2	5	L1

PART- B (5 x 13 = 65 Marks)

Q. No	Questions	Marks	CO	BL
11(a)(i)	A continuous random variable X has a probability density function given by $f(x) = \begin{cases} kxe^{-\lambda x} & ; x \geq 0, \lambda > 0 \\ 0 & ; otherwise \end{cases}$. Determine the constant k. Obtain the mean and variance of X.	6	1	L3
(ii)	Derive moment generating function for Binomial distribution and hence find its mean and variance.	7	1	L3
	OR			
11(b)(i)	In a certain industrial facility, accidents occur infrequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other. (a) What is the probability that in any given period of 400 days there will be an accident on one day? (b) What is the probability that there are at most three days with an accident?	6	1	L3
(ii)	The mileage (in thousands of miles) that car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000 miles. Find the probabilities that one of these tires will last (a) at least 20,000 miles, (b) at most 30,000 miles.	7	1	L3
12(a)(i)	Determine the value of c that makes the function $f(x,y) = c(x+y)$ a joint probability mass function over the nine points with $x = 1,2,3$ and $y = 1,2,3$. Determine (a) value of c, (b) $P(X = 1, Y < 4), P(X < 2, Y < 2)$.	6	2	L3
(ii)	Let X and Y are normally distributed independent random variables with mean 0 and variance σ^2. Find the joint probability density function of $R = \sqrt{x^2 + y^2}$ and $\theta = tan^{-1}\left(\frac{y}{x}\right)$.	7	2	L3
	OR			
12(b)	The joint probability density function of a two dimensional random variables is given by $f(x,y) = \frac{1}{3}(x+y);\ 0 \leq x \leq 1,\ 0 \leq y \leq 2$. Find the correlation coefficient.	13	2	L3
13(a)(	To compare two brands of cigarettes, Brand A and Brand B, for their tar content, a sample of 60 was inspected from Brand A and a sample of 40 from Brand B. The results of the tests are summarized as follows: Brand A: $\bar{x}_1 = 15.4\,, s_1^2 = 3$ Brand B: $\bar{x}_2 = 16.8\,, s_2^2 = 4$ At 5 percent level of significance, do the two brands differ in their mean tar content?	7	3	L4
(ii)	A random sample of 15 adults living in a small town were selected to estimate the proportion of voters favoring a certain candidate for mayor. Each individual was also asked if he or she was a college graduate. By letting Y and N designate the responses of "yes" and "no" to the education question, the following sequence was obtained: $N\ N\ N\ N\ Y\ Y\ N\ Y\ Y\ N\ Y\ N\ N\ N\ N$ Use the runs test at the 0.01 level of significance to determine if the sequence supports the contention that the sample was selected at random.	6	3	L4

OR

13(b)(i) The following table shows the amounts of carbohydrates (in grams) per serving of two varieties of canned peaches.

Variety A : 32 32 27 38 29

Variety B : 27 28 33 30 26 29 28

Based on this sample data, at the 0.05 level of significance, are the two varieties significantly different in their true mean carbohydrates contents?

Marks: 7 | CO: 3 | L4

(ii) A study is conducted to compare the lengths of time required by men and women to assemble a certain product. Past experience indicates that the distribution of times for both men and women is approximately normal but the variance of the times for women is less than that for men. A random sample of times for 11 men and 14 women produced the following data:

Men $: n_1 = 11 , s_1 = 6.1$

Women: $n_2 = 14, s_2 = 5.3$

Test the hypothesis that $\sigma_1^2 = \sigma_2^2$ against the alternative that $\sigma_1^2 > \sigma_2^2$.

Marks: 6 | CO: 3 | L4

14(a) A company appoints 4 salesmen A, B, C, D and observes their sales in three seasons summer, winner and monsoon. The figures (in lakhs of Rupees) are given in the following table: (Use $\alpha = 0.05$)

	Salesmen			
Season	A	B	C	D
Summer	36	36	21	35
Winter	28	29	31	32
Monsoon	26	28	29	29

Carry out an analysis of variance.

Marks: 13 | CO: 4 | L4

OR

14 (b) In a 5×5 Latin square experiment, the data collected is given in the matrix below. Yield per plot is given in quintals for five different cultivation treatments A, B, C, D and E. Perform the analysis of variance. (Use $\alpha = 0.05$)

A 48	E 66	D 56	C 52	B 61
D 64	B 62	A 50	E 64	C 63
B 69	A 53	C 60	D 61	E 67
C 57	D 58	E 67	B 65	A 55
E 67	C 57	B 66	A 60	D 57

Marks: 13 | CO: 4 | L4

15 (a) The following data give the measurements of 20 samples, each of size 3, is the production process taken in the interval of 2 hours. Draw $\bar{X}$ chart and R chart. Hence check whether the statistical process in under the control or not?

Sample No.	1	2	3	4	5	6	7	8	9	10
Observations	36	30	33	35	33	32	27	32	32	36
	33	34	32	30	31	34	36	36	33	40
	34	31	29	34	33	33	35	41	39	37

Sample No.	11	12	13	14	15	16	17	18	19	20
Observations	20	30	34	36	38	33	36	35	36	34
	30	32	35	39	33	43	39	34	33	33
	33	38	30	37	34	35	37	31	37	31

Marks: 13 | CO: 5 | L4

OR

(ii)	The data given below are the number of defectives in 20 samples of size 50. Construct a $p - chart$ and comment on the results:	13	5	L4

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of defectives	8	6	5	7	2	5	3	8	4	4
Sample No.	11	12	13	14	15	16	17	18	19	20
No. of defectives	3	1	5	4	4	2	3	5	6	3

PART- C (1 x 15 = 15 Marks)

Q. No	Questions	Marks	CO	BL
16(i)	The IQs of 600 applicants to a certain college are approximately normally distributed with a mean of 115 and a standard deviation of 12. If the college requires an IQ of at least 95, how many of these students will be rejected on this basis of IQ, regardless of their other qualifications?	5	1	L6
(ii)	The contingency table below summarizes the results obtained in a study conducted by a research organization with respect to the performance of four competing brands of toothpaste among the users.	10	3	L5

	Brand A	Brand B	Brand C	Brand D
No cavities	9	13	17	11
One to five cavities	63	70	85	82
More than five cavities	28	37	48	37

Test for hypothesis that the incidence of cavities is independent of the brand of toothpaste used.

4/4 End